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A NOTE ON CONSTRUCTION OF GRAECO-LATIN SRUARE OF ORDER 2n + 1

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SUMMARY

A simple method of constructing a Graeco-Latin square (equivalently, a pair of orthogonal Latin squares) of any odd order ($\geqslant 3$) is given.

Introduction

The construction of a set of mutually orthogonal Latin squares has been an active area of research, especially for orders which are not prime powers. A fairly good description of advances in this area may be found, e.g. in Denes and Keedwell [1], and Hall ([2]; Chapter 13). In this note, we propose a simple method of constructing a pair of orthogonal Latin squares (equivalently, a Graeco-Latin square) of any odd order. It is shown that it is possible to construct a pair of orthogonal Latin squares of any odd order by 'developing' two initial rows.

2. The Result

THEOREM : *A Graeco-Latin Square (equivalently a pair of orthogonal Latin Square) of order* $2n + 1$ *can be obtained by developing the following two initial rows and mod* $(2n + 1)$

$$R_1 : \quad 0 \quad 1 \quad 2 \quad \ldots \quad 2n$$
$$R_2 : \quad 0 \quad 2n \quad 2n-1 \quad \ldots \quad 1$$

Proof. Clearly, by developing R_1 and R_2, mod $(2n + 1)$, we get two

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Latin squares. Let the (i, j)th position of the first latin square, say L_1, obtained by developing tne initial row R_1 have the symbol k. Then,

$$k = (i + j - 2) \bmod (2n + 1).$$

Also if the (i, j)th position of the second latin square, say L_2 obtained by developing the initial row R_2, has the symbol m, then

$$m = (2n + 1 + i - j) \bmod (2n + 1)$$

Now k and m are equal if

$$i + j - 2 = (2n + 1 + i - j) \bmod (2n + 1)$$

i.e. $$2(j - 1) = 0 \bmod (2n + 1)$$

But this is true only if $j = 1$. Hence $k = m$ only in the fisrt column and no where else.

After we have got a pair of numbers k and m such that k occurs in (i, j)th cell of L_1 and m occurs in the (i, j)th cell of L_2, let it be possible that the same pair of numbers k and m occurs in (i', j')th cell of L_1 and L_2 respectively. In this situation we have,

$$k = (i + j - 2) \bmod (2n + 1)$$

$$m = (2n + 1 + i - j) \bmod (2n + 1) \qquad (1)$$

and also

$$k = (i' + j' - 2) \bmod (2n + 1)$$

$$m = (2n + 1 + i' - j') \bmod (2n + 1) \qquad (2)$$

Thus, from (1) we have,

$$k - m = 2(j - 1) \bmod (2n + 1) \qquad (3)$$

and from (2),

$$k - m = 2(j' - 1) \bmod (2n + 1) \qquad (4)$$

Hence from (3) and (4) we have

$$2(j - j') = 0 \bmod (2n + 1)$$

But this is possible only if $j = j'$ since $1 \leqslant j, j' \leqslant 2n + 1$. Similarly it can be shown by using the sum $k + m$ that the above contention is true only if $i = i'$. Hence the pair (k, m) cannot occur more than once. Accordingly by superimposing L_1 and L_2 we always get a Graeco-Latin Square of order $2n + 1$.

REFERENCES

[1] J. Denes and Keedwell, A. D. (1974): *Latin Squares and their Applications.* Academic Press, New York.

[2] Hall, M. Jr. (1986): *Combinatorial Theory.* Wiley, New York.